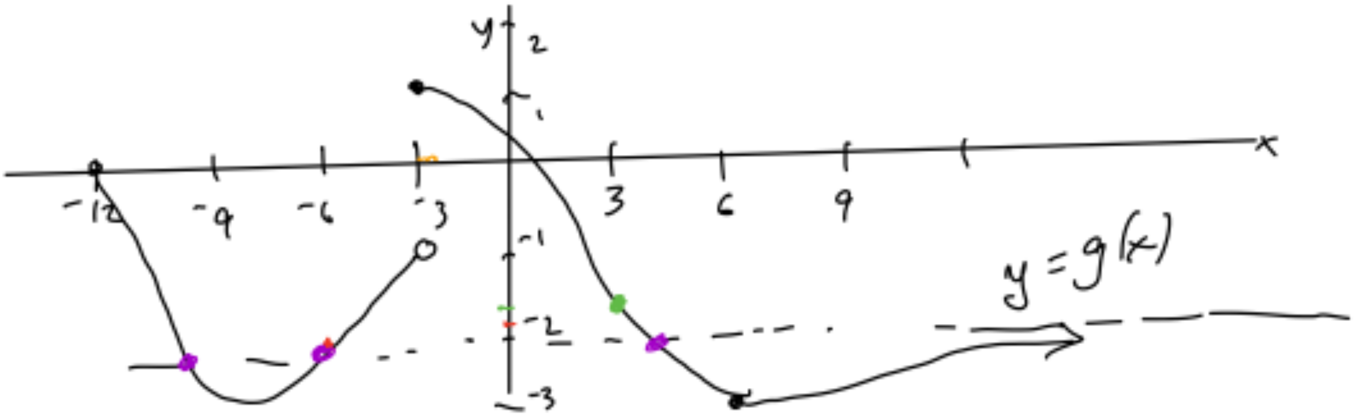


Questions:

① Estimate $g(3) \approx -1.71$

② Estimate $g(-6) \approx -1.9$

③ Estimate $g(-3) \approx 1.06$

④ Solve $g(x) = -2$. $x = -9.9, -6.1, 4$

⑤ At which x is g not continuous? $x = -3$.

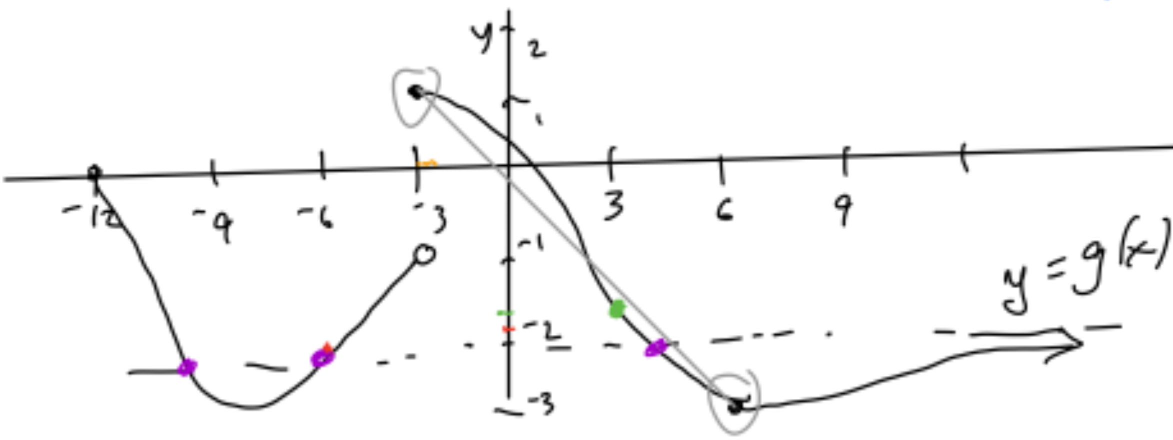
⑥ $\lim_{x \to -3^+} g(x) = 1.06$

as x approaches -3 from the + side

⑦ $\lim_{x \to -3^-} g(x) = -.84$

⑧ $\lim_{x \to \infty} g(x) = -2$

(horiz asymptote at -2!)



⑨ What is the average rate of change of $g(x)$ between $x = -3$ and $x = 6$?

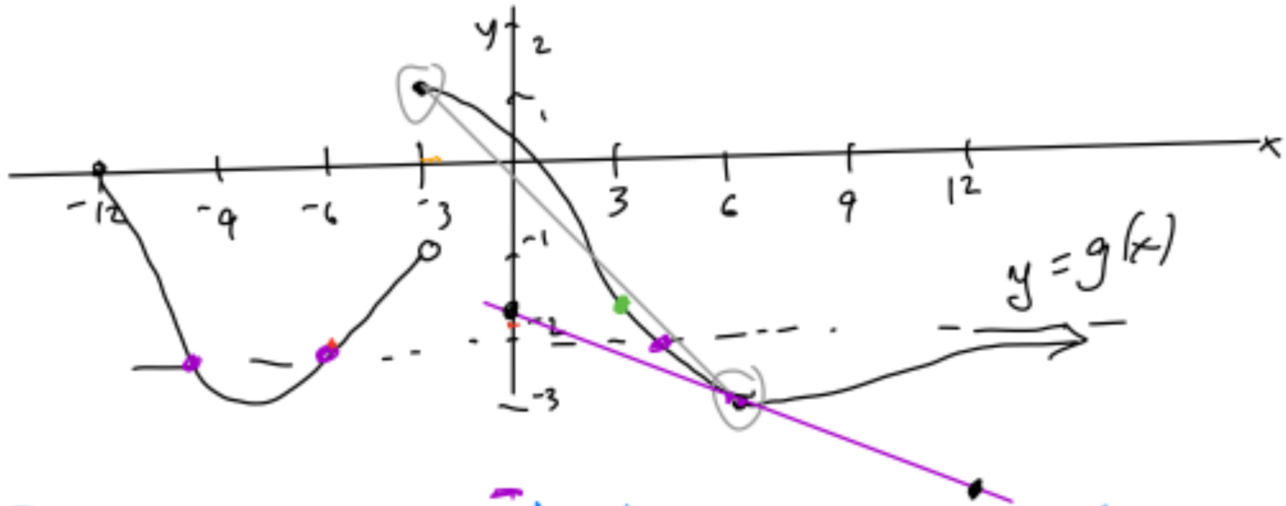
$$\left(\begin{matrix}\text{average rate}\\ \text{of change}\end{matrix}\right) = \frac{\Delta \text{ function}}{\Delta \text{ variable}} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

= slope of secant line connecting points where $x = -3$ & $x = 6$.

$$= \frac{-3 - 1.06}{6 - (-3)} = \frac{-4.06}{9} = -0.44$$

Notice: the average rate of change is negative

→ the function decreases in moving from $x=-3$ to $x=6$.



(10) What is the instantaneous rate of change of g at $x=6$? (the derivative of g at $x=6$)

= slope of the tangent line to the graph of $y=g(x)$ at $x=6$.

$$= \frac{\Delta y}{\Delta x} = \frac{-4.07-(-1.67)}{12-0} = \frac{-4.07+1.67}{12} = \frac{-2.4}{12} = \boxed{-.20}$$

Example:

(1.4c) Rewrite as a sum of power fcns.

$$\frac{(\sqrt{x}-3)^3}{x} = \frac{(\sqrt{x})^3 - 3(\sqrt{x})^2\cdot 3 + 3\cdot(\sqrt{x})(3)^2 - 3^3}{x}$$

$x>0$

$$= \frac{x^{3/2} - 9\cdot x + 27x^{1/2} - 27}{x^1}$$

$$= \boxed{x^{1/2} - 9 + 27x^{-1/2} - 27x^{-1}}$$

1
1 1
1 2 1
1 3 3 1

1.7d Simplify $\frac{2\cos^3(x)\sec(x)}{\tan(x)} = \frac{2\cos^3(x)\frac{1}{\cos(x)}}{\frac{\sin(x)}{\cos(x)}}$

$$= \frac{2\cos^2(x)}{\frac{\sin(x)}{\cos(x)}} = 2\cos^2(x)\cdot\frac{\cos(x)}{\sin(x)}$$

$$= \boxed{\frac{2\cos^3(x)}{\sin(x)}} = 2\cot(x)\cos^2(x)$$

Limits

f – some function

We say that the limit of $f(x)$ as x approaches a is L,

i.e. $\lim_{x\to a} f(x) = L$,

if as x gets very close to a (but not equal to a), then $f(x)$ gets very close to L.

Examples

① $\lim_{x\to 4} \frac{x^2-1}{5x} = \frac{15}{20} = \boxed{\frac{3}{4}}$

$x^2-1 \to 15$, $5x \to 20$

② $\lim_{c\to 4} \frac{c-4}{2c^2-7c-4} = \lim_{c\to 4} \frac{\cancel{c-4}}{(2c+1)\cancel{(c-4)}} = \lim_{c\to 4} \frac{1}{2c+1} = \boxed{\frac{1}{9}}$

$c-4 \to 0$, $2c^2-7c-4 \to 0$

$32-28-4=$

$2c+1 \to 9$

③ $$\lim_{x \to 2} \frac{2^x - 4 \to 0}{\sin(x-2) \to 0} = 2.77258\ldots$$

↑ check by numerical calculation.

Type some Sage code below and press Evaluate.

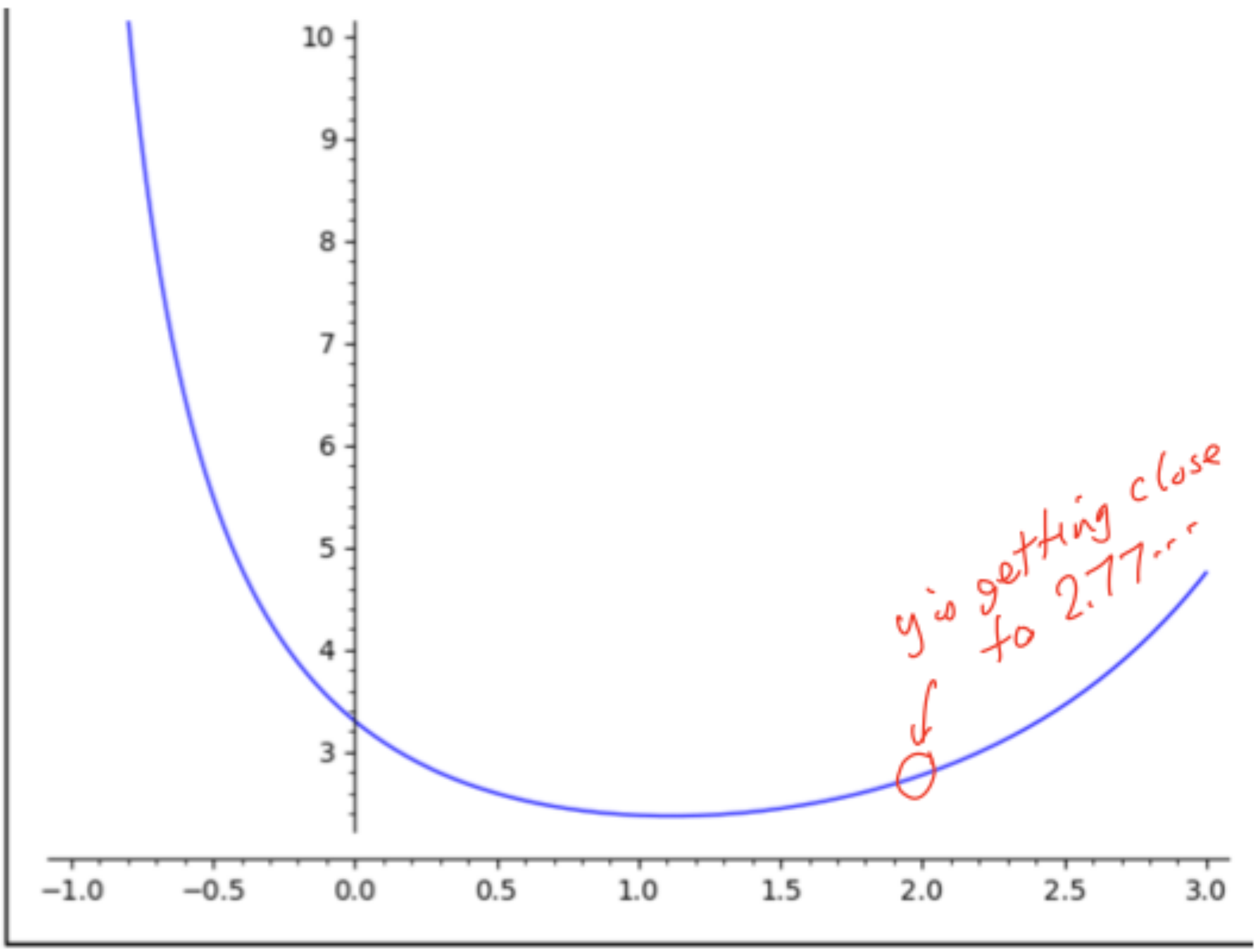
```
f(x) = (2^x-4)/sin(x-2)
xs = [2.1,1.9,2.03,2.00006,1.999997]
# the above is a list of numbers
ans = [f(b) for b in xs]
show(ans)
show(limit(f(x),x=2))
show(4*log(2).n())
```

Evaluate

[2.87572898722678, 2.68315001980132, 2.80203704760290, 2.77264637906103, 2.77258583

$4\log(2)$

2.77258872223978



④ $\lim_{x\to 1} \frac{2^x - 4}{\sin(x-2)} = \frac{-2}{\sin(-1)} = \frac{-2}{-\sin(1)} = \boxed{\frac{2}{\sin(1)}}$

$2^x - 4 \to -2$

$\sin(x-2) \to \sin(-1)$

⑤ $\lim_{y\to \frac{1}{2}} \frac{6y^2 - y - 1}{4y^2 - 4y + 1}$

$= \lim_{y\to\frac{1}{2}} \frac{(3y+1)(2y-1)}{(2y-1)^2}$

$= \lim_{y\to\frac{1}{2}} \frac{3y+1}{2y-1}$

$3y+1 \to 2.5$, $2y-1 \to 0$

$\lim_{y\to\frac{1}{2}^-} \frac{3y+1}{2y-1} = -\infty$ (y: .499; $3y+1 \to 2.5$; $2y-1 \to 0$ negative)

$\lim_{y\to\frac{1}{2}^+} \frac{3y+1}{2y-1} = +\infty$ (y: .501; $3y+1 \to 2.5$; $2y-1 \to 0$ positive)

$\boxed{\text{limit DNE}}$ Does Not exist.

⑥ $\lim_{y\to\infty} \frac{6y^2 - y - 1}{4y^2 - 4y + 1}$

$= \lim_{y\to\infty} \left(\frac{3y+1}{2y-1}\right)$

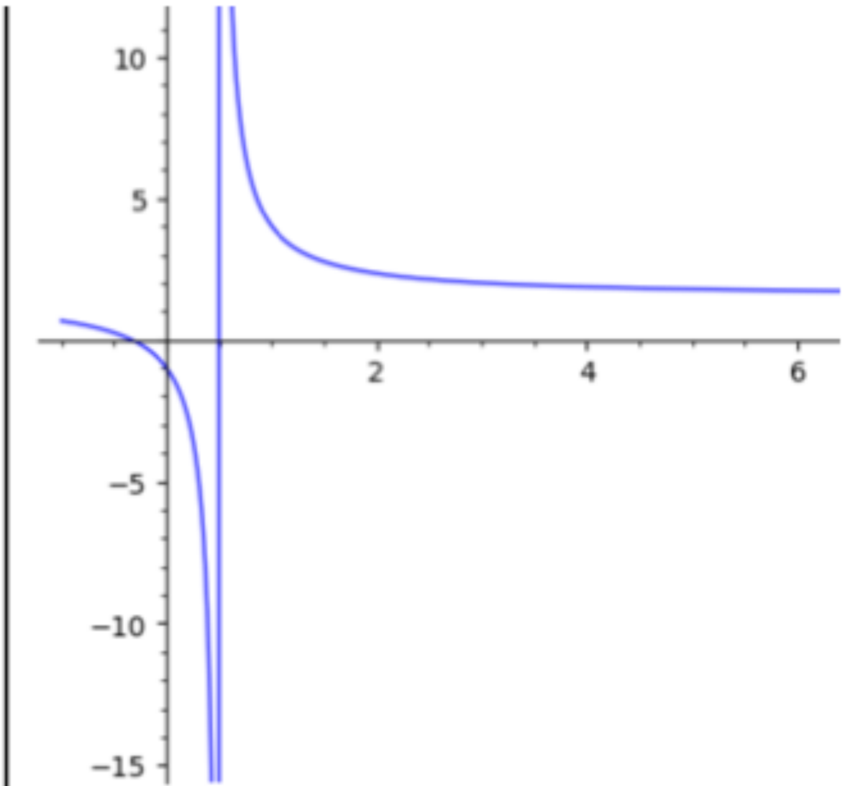
$= \lim_{y\to\infty} \frac{\cancel{y^2}\left(6 - \frac{1}{y} - \frac{1}{y^2}\right)}{\cancel{y^2}\left(4 - \frac{4}{y} + \frac{1}{y^2}\right)}$

$\frac{1}{y} \to 0$, $\frac{1}{y^2} \to 0$, $\frac{4}{y} \to 0$, $\frac{1}{y^2} \to 0$

$= \frac{6}{4} = \boxed{\frac{3}{2}}$

Type some Sage code below and press Evaluate.

```
g(y)=(6*y^2-y-1)/(4*y^2-4*y+1)
show(plot(g(y),y,-1,10,ymin=-15,ymax=15))
show(limit(g(y),y=1/2))
show(limit(g(y),y=oo))
```



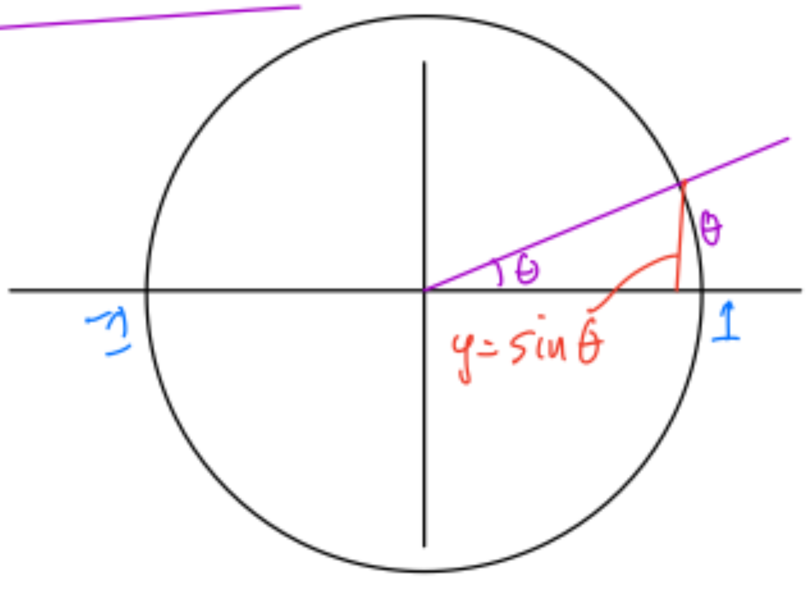
∞

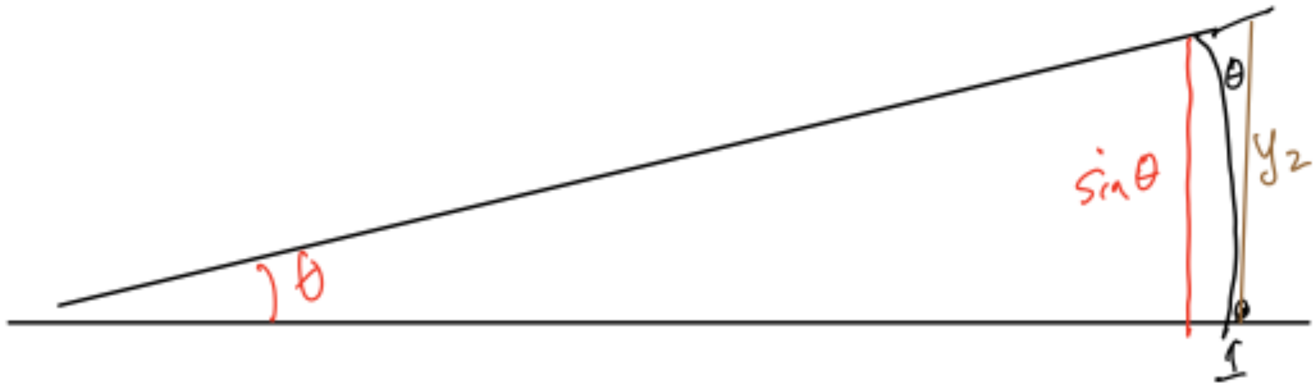
$\frac{3}{2}$

Some Special Limits :

① $\lim\limits_{\theta \to 0} \frac{\sin(\theta)}{\theta}$.

[θ is measured radians]



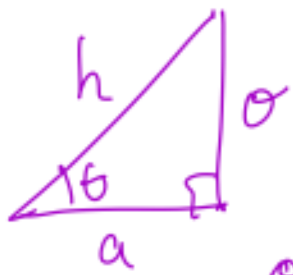


$$\frac{y_2}{1} = \tan\theta$$

for small θ,

$$\sin\theta < \theta < y_2 = \tan\theta$$

sandwich.



$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

$\tan\theta = \frac{o}{a}$

$$\sin\theta < \theta < \frac{\sin\theta}{\cos\theta}$$

divide by $\sin\theta$ everywhere, $\theta > 0$

$$\frac{\sin\theta}{\sin\theta} < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

flip

$$1 > \frac{\sin\theta}{\theta} > \cos\theta$$

$\theta \to 0$

$\downarrow$ 1 $\quad$ $\downarrow$ must $\to 1$ $\quad$ $\downarrow$ 1

$$\therefore \boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1}$$